Carnegie Mellon University
Tepper School of Business

47-827 Current Issues in GMM Estimation and Inference Spring 2006
(6 units)

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Class Meetings: Tuesday & Thursday
10:30 – 12:20 rm 318 GSIA Building

OVERVIEW

The past 10 years has seen a number of advances beyond traditional two-step GMM estimation. These advances have focused on providing moment-based estimators and hypothesis tests with more accurate approximation to the sampling distributions. Examples include: continuous updated GMM (Hansen, Heaton and Yaron 1996, JBES), the empirical likelihood estimator (Imbens et al., 1998, Econometrica) and the exponential tilting estimator of (Kitamura and Stutzer 1997, Econometrica), Bootstrap GMM (Hall and Horowitz 1996, Econometrica), GMM with Weak Instruments (Stock and Wright 2000, Econometrica), GMM with the Empirical Saddlepoint (Sowell, 2006). For empirical work these estimators are preferred over the traditional two-step GMM estimator.

This course will present (*i*) the motivation for these advances, (*ii*) a summary of published results and (*iii*) current research.

GRADING

Your course grade will be determined by a final paper. Select an empirical problem that can be estimated by GMM and collect the appropriate data.

I suggest you not be too ambitious with your empirical example. It would be best if your model contained two or three parameters and had four or five moment conditions. It will be difficult to develop intuition with a more complicated model. In the empirical work that I present in class, I will typically have two moment conditions and one parameter. After you have developed intuition, you will be ready for more advanced empirical problems, i.e. your summer papers, dissertations, etc.

Your final paper will consist of your explanation of the Economics of the problem, a brief review of the previous research results and a report of your estimation and inference for this model using three different procedures.

1. Traditional two-step GMM.
2. A one-step GMM procedure such as empirical likelihood, exponential tilting or continuously updated GMM.
3. Empirical Saddlepoint approximation.

You should compare and contrast the results. You should also include

- A general description of the data.
- A plot of the series.
- Any problems with the series. (Was the model stable over the entire period? Are there any outliers?)
- Related to the previous item, explanations of any simplifying assumptions used in modeling the series.

I will provide MATLAB programs that implement each of these three different estimation techniques. You will need to rewrite the programs to incorporate your moment conditions and data. We will use class time to talk about how the MATLAB programs work.

I am comfortable with you helping each other with the computer work. However, each student should select a different empirical problem.

SEQUENCE OF TOPICS IN THE LECTURES

1. Traditional two-stage GMM. Introduce notation. Refresh the basic consistency and asymptotic normality arguments (first order asymptotics). Introduce the identifying subspace and overidentifying subspace. Hansen's test for overidentifying restrictions and its asymptotic distribution.

   Note the basic structure of GMM estimation and inference is concerned with three terms
   $$M(\theta)W_N(\theta)G_N(\theta).$$

   Note that the small sample distribution are often not well approximated by the first order asymptotic results.

   Think of applied work as obtaining estimates for θ, $M(\theta)$ and $W_N(\theta)$ to be plugged into the formulas for the asymptotic distribution.

2. A simple MATLAB program to perform two-step GMM.

3. Introduction to one-step GMM estimators. Think of these as providing alternative estimates for θ, $M(\theta)$ and $W_N(\theta)$ to be plugged into the formulas for the asymptotic distribution. Three different ways to test the validity of the overidentifying restrictions.

4. A simple MATLAB program for a one-step estimator.

5. The Asymptotic distribution for one-step GMM estimators using the general $h(\bullet)$ function.

6. The calculation of the higher order bias for one-step GMM estimators. This then gives the bias corrected estimators.

7. The calculation of the higher order MSE for one-step GMM estimators. This gives a way to rank estimator with the same higher order bias.

8. Edgeworth expansions as a higher order generalization of the first order expansion that gives the Central Limit Theorem. Cumulant Generating Function. The Moment Generating Function.

9. Problems with Edgeworth expansions. The Bootstrap. How the Edgeworth expansion shows the bootstrap's faster order of convergence.

10. The S-sets for GMM with weak instruments.

11. Introduction to the saddlepoint approximation. The application of an Edgeworth expansion for every point in the extended parameter space. The higher rate of convergence for the saddlepoint approximation. The importance of normalization to (possibly) increase the rate of convergence.

12. MATLAB program to perform a saddlepoint approximation for the sample mean of a known distribution.

13. The saddlepoint approximation for parameter estimates determined by estimation equations with a known distribution.

14. The empirical saddlepoint approximation. The saddlepoint approximation for parameter estimates determined by estimation equations with an unknown distribution.

15. MATLAB program to perform the empirical saddlepoint approximation.

SOME BASIC REFERENCES

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