Carnegie Mellon University
Graduate School of Industrial Administration

47-819 (6 units)	Estimation of Continuous Time Models	Spring 2003

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Class Meetings: Tuesday & Thursday
10:30 – 12:20 rm 318 GSIA Building

Objectives: what students should learn.

1. The ability to take the parameters from an estimated model and use them to address a practical problem.
2. The ability to articulate the basic procedures for estimating continuous time models.
3. The restrictions on the models that permit different estimation procedures.
4. The properties of the estimates from the different estimation procedures.
5. The ability to judge the adequacy of the fit of estimated models.
6. An understanding of the basic steps involved in obtaining parameter estimates from these different procedures.
7. An understanding of the difficulties in doing applied work, so that you can better understand the limitations of these models.
8. An understanding of how parameter and model uncertainty affect the prices for derivative securities.

GRADING

Your final course grade will be determined by a paper that you will write during the Mini. You will apply one of the estimation techniques presented in the lectures to a data series. You will report the parameter estimates and explain the impact of the estimated parameters and models on the pricing of a derivative security.

The papers will be broken down into two parts. A midterm summary will be due on April 18 and should include:

1. A general description of the data.
2. A plot of the series.
3. A brief description of the market(s) which generated the series.
4. Any problems with the series. (Was the model stable over the entire period? Are there any outliers?)
5. Related to the previous item, explanations of any simplifying assumptions used in modeling the series and the pricing problem. (How are weekends and holidays treated? How are outliers treated?)
6. A brief description of the derivative security that you will be considering and an explanation of how the price will be determined (eg. simulation, numerical solution of a PDE, etc.).

The final paper will be due on May 9. The final paper will include a model estimated by one of the procedures presented in the class. For the model, you need to report the implied estimate of the price of the derivative security.

OVERVIEW

We will start with estimation by maximum likelihood. For some models there exists closed forms for the transition densities, which is sufficient to determine the likelihood function. This can be maximized to obtain parameter estimates and standard errors. Two examples will be presented; one is the estimation of the geometric Brownian motion model, and the other is the estimation of the Ornstein-Uhlenbeck process. A nice paper for this general idea is:

Lo, Andrew (1988), "Maximum Likelihood Estimation of Generalized Itô Processes with Discretely Sampled Data" **Econometric Theory** 4, 231-247.

While presenting the material on maximum likelihood estimation, we will cover a short section on numerical optimization of nonlinear function.

After maximum likelihood, we will consider estimation by the Generalized Method of Moments. The classical papers on this topic are:

Hansen, Lars Peter (1982), "Large Sample Properties of Generalized Method of Moments estimators" **Econometrica** 50, 1029-1054.

and

Hansen, Lars Peter and Kenneth J. Singleton (1982), "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models" **Econometrica** 50, 1269-1286.

As an application of the GMM framework and as an introduction to Euler approximation of a continuous time stochastic processes, we will consider the model estimated in

Chan, K. C., G. Andrew Karolyi, Francis A. Longstaff and Anthony B. Sanders (1992) "An Empirical Comparison of Alternative Models of the Short-Term Interest Rate" **The Journal of Finance** 48, 1209-1227.

Note that an approximation scheme (such as the Euler approximation) will be needed in the Efficient Method of Moments estimation technique of Gallant and Tauchen.

We will then consider the Efficient Method of Moments technique of Gallant and Tauchen. This procedure is basically a GMM estimation procedure where the moments of the model are obtained by simulation, (hence the need for an approximation technique). The idea is to estimate some auxiliary model with the observed data. Then select parameter estimates so that the estimated auxiliary model is close to the auxiliary model estimated by the observed data. The papers related to this technique are:

Gallant, A. Ronald and George Tauchen, (1997) "Estimation of Continuous Time Models of Stock Returns and Interest Rates" **Macroeconomic Dynamics** 1, 135-168.

George Tauchen, "New Minimum Chi-Square Methods in Empirical Finance" **Advances in Economics and Econometrics: Theory and Applications**, Vol. III, Econometric Society Monographs No. 28, David M. Kreps and Kenneth F. Wallis (eds.), Cambridge University Press, 1997, pp. 279-317.

and

Gallant, A. Ronald and Jonathan R. Long, (1997) "Estimating Stochastic Differential Equations Efficiently by Minimum Chi-Square" **Biometrika** 84, 125-141.

We will then consider the nonparametric approach of Yacine Aït-Sahalia. The basic idea is to derive a *relationship* between the diffusion function and the drift and the marginal density for the stochastic process. The drift is estimated by nonlinear least squares, and the marginal density is estimated by a nonparametric kernel estimator. The above *relationship* is then used to identify the diffusion function. The papers related to this technique are:

Aït-Sahalia, Yacine (1996) "Nonparametric Pricing of Interest Rate Derivative Securities" **Econometrica** 64, p527-560

and

Aït-Sahalia, Yacine (1996) "Testing Continuous-Time Models of the Spot Interest Rate" **The Review of Financial Studies** 9, p385-426.

We will then consider work related to the "back to the future" approach of Hansen and Scheinkman. This technique is basically a GMM technique. The idea is to use semigroup properties to derive the moment conditions used in estimation. The papers related to this technique are

Hansen, Lars Peter, José Scheinkman (1995) "Back to the Future: Generating Moment Implications for Continuous-Time Markov Processes" **Econometrica** 63, p767-804

Conley, et al. (1996) "Short-Term Interest Rates as Subordinated Diffusions", **Review of Financial Studies Paper** 10:3, Fall 1997, 525-577.

and

Hansen, Lars Peter, José Scheinkman and Nizar Touzi (1998) "Spectral Methods for Identifying Scalar Diffusions," **Journal of Econometrics** 86, 1-32.

We will then consider the an approximation to MLE is presented in

Aït-Sahalia, Yacine (2002) "Maximum Likelihood Estimation of Discretely Sampled Diffusions: A Closed-Form Approximation Approach," **Econometrica** 70:1, 223-262.

and

Aït-Sahalia, Yacine (2002) "Closed-Form Likelihood Expansions for Multivariate Diffusions," Department of Economics, Princeton University, working paper.

The course will conclude with a couple of papers that show how the empirical characteristic function is used to estimate continuous time models:

Singleton, Kenneth J. (2001) "Estimation of Affine Asset Pricing models using the empirical Characteristic function," **Journal of Econometrics** 102, 111-141.

and

Carrasco, Marine, Mikhail Chernov, Jean-Pierre Florens and Eric Ghysels (2002) "Efficient estimation of jump diffusions and general dynamic models with a continuum of moment conditions," Department of Economics, University of Rochester, working paper.